Game Theory, Spring 2024

Problem Set # 3

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Due Mar 20 at 5:15 PM

Exercise 1

1. Complete the proof of Proposition 1 from Lecture #4.
2. For the second-price sealed-bid auction considered in Lecture #4 find another Bayesian Nash equilibirum.

Exercise 2

Suppose $I \geq 2$ bidders participate in a second-price sealed-bid auction with a minimum bid r. Bidders can choose to *not participate* or place a bid above $r > 0$, i.e. the action space of each bidder is $\{\emptyset\} \cup [r, +\infty)$, where $\emptyset$ denotes the non-participation action. If several bidders participate, then the highest bidder gets the object and pays the second-highest bid, and everybody else pays nothing. Ties are broken uniformly at random. If only one bidder participates, then this bidder wins and pays r. Non-participating bidders pay nothing. Bidder i assigns value V_i to the object. V_i is distributed on $[0, 1]$ according to F, independently and identically across bidders. F has a continuous density f and full support. Bidder i knows her own value, but does not know the values of her competitors.

1. Formally define this auction as a Bayesian game.
2. Show that there is a Bayesian Nash equilibrium in dominant strategies, in which

each bidder plays β given by:

$$\beta(v_i) = \begin{cases} v_i & \text{if } v_i \geq r, \\ \emptyset & \text{otherwise.} \end{cases}$$

3. Compute expected revenue (as a function of r) achieved in this equilibrium.
4. Suppose $F(x) = x$ (i.e. V_i is uniform on $[0,1]$ for all i), find the revenue-maximizing minimum bid and compute optimal revenue.

Exercise 3

Suppose $I \geq 2$ bidders participate in a glum-loser auction. The highest bidder gets the object and pays nothing, everybody else pays their own bid. Ties are broken uniformly at random. Bidder i assigns value V_i to the object. V_i is distributed on $[0,1)$ according to F, independently and identically across bidders. F has a continuous density f and full support. Bidder i knows her own value, but does not know the values of her competitors.

1. Formally define this auction as a Bayesian game.
2. Find a symmetric Bayesian Nash equilibrium in strictly increasing strategies.
3. Compute expected revenue achieved in this equilibrium (Hint: define revenue as $R^* \equiv \sum_{i=1}^{I} \beta(V_i) - \max\{\beta(V_1), \ldots, \beta(V_I)\}$).
4. Suppose $F(x) = x$ (i.e. V_i is uniform on $[0,1)$ for all i), evaluate expected revenue in this case.

Exercise 4

Suppose $I = 2$ bidders participate in a first-price sealed-bid auction with *common values*. The highest bidder gets the object and pays her own bid, everybody else pays nothing. Ties are broken uniformly at random. Each bidder gets a signal S_i, which is uniformly distributed on $[0,1]$, independently and identically across bidders. Bidder i knows her own signal realization, but does not know the signal realization of her competitor. Bidder i's value for the object is equal to the sum of all signals, i.e. $V_i = \sum_{j=1}^{I} S_j$.

1. Formally define this auction as a Bayesian game.
2. Suppose β is strictly increasing and continuosly differentiable, and $\beta(0) = 0$. Compute the expected utility of bidder i who chooses to bid b_i and whose signal realization is s_i (Hint: conditional on winning, bidder i values the object at $\mathbb{E}[V_i | b_i \geq \beta(S_{-i}),\ S_i = s_i]$).
3. Find a symmetric Bayesian Nash equilibrium in strictly increasing strategies.
4. Compute expected revenue achieved in this equilibrium.