Game Theory, Spring 2024

Problem Set # 2

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Due Mar 6 at 5:15 PM

Exercise 1

Compute expected equilibrium payoffs of both firms in the Cournot duopoly with incomplete information discussed in the lecture.

Exercise 2

Two firms compete in quantities. The inverse demand function is $P(q_1, q_2) = \max\{\alpha - q_1 - q_2,\ 0\}$, where q_1 and q_2 are quantities set by Firm 1 and Firm 2 respectively, and α is a parameter that determines the demand conditions. Demand can be *high* ($\alpha = \alpha_H$) with probability π_H, or *low* ($\alpha = \alpha_L$) with probability π_L. Naturally, $\alpha_H > \alpha_L > 0$. Firm 1 knows the demand conditions (i.e. the value of α) but Firm 2 does not. Both firms have *zero* marginal costs.

1. Formally define this strategic situation as a Bayesian game.
2. Find an interior Bayesian Nash equilibrium in pure strategies and derive conditions on the parameter values that ensure its existence.
3. Compute expected equilibrium payoffs of both firms.

Exercise 3

Two firms supply differentiated products and compete in prices. The demand for Firm i's product is given by $D_i(p_i, p_{-i}) = \max\{\alpha - p_i + p_{-i},\ 0\}$, where p_i and

p_{-i} are prices set by Firm i and Firm $-i$ respectively, and $\alpha > 0$ is known by both firms. Each firm's marginal cost can be low (c_L, with probability π_L) or high (c_H, with probability π_H), independently of the other firm. A firm knows its own marginal cost, but does not know the marginal cost of its competitor.

1. Formally define this strategic situation as a Bayesian game.

2. Find a symmetric interior Bayesian Nash equilibrium in pure strategies and derive conditions on the parameter values that ensure its existence.

3. Compute expected equilibrium payoffs of both firms.

Exercise 4

Two firms supply non-differentiated products and compete in prices. The demand for Firm i's product is given by:

$$D_i(p_i, p_{-i}) = \begin{cases} 1 & \text{if } p_i < p_{-i} \text{ and } p_i < v, \\ \frac{1}{2} & \text{if } p_i = p_{-i} < v, \\ 0 & \text{otherwise,} \end{cases}$$

where p_i and p_{-i} are prices set by Firm i and Firm $-i$ respectively, and v is the consumers' willingness-to-pay. Each firm's marginal cost can be low (c_L, with probability π_L) or high (c_H, with probability π_H). A firm knows its own marginal cost, but does not know the marginal cost of its competitor. *We now assume that $c_H > v > c_L$.*

1. Formally define this strategic situation as a Bayesian game.

2. Show that there is no symmetric Bayesian Nash equilibrium in pure strategies.

3. Find a symmetric Bayesian Nash equilibrium in mixed strategies *(We will cover the material on mixed strategies in the beginning of Lecture #3).*