First Best Implementation with Costly Information Acquisition*

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Abstract

We study mechanism design with flexible but costly information acquisition. There is a principal and four or more agents, sharing a common prior over a set of payoff-relevant states. The principal proposes a mechanism to the agents who can then acquire information about the state by privately designing a signal device. As long as it is costless for each agent to acquire a signal that is independent from the state, there exists a mechanism which allows the principal to implement any social choice rule at zero information acquisition cost to the agents.

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1 Introduction

In most mechanism design problems, there is a collection of agents who have exogenously given private information, and there is a principal who desires to implement a social choice rule by designing a mechanism which incentivizes the agents to reveal their information.

In many practical problems, however, the agents' private information is often a consequence of their own (possibly costly) *information acquisition*. For example, bidders in an oil-tract auction (Wilson, 1969) may conduct test drills; bidders in a spectrum auction may conduct market research; voters in a presidential election may investigate the candidates' past political activities; members of a hiring committee may study the job applicant's background in order to see whether he is fit for the job.

Importantly, in such situations, a mechanism in place does not only affect each agent's incentive to report the acquired information truthfully, but also affects his choice of *what kind of information* to acquire. In this sense, the properties of desirable mechanisms could potentially be very different from those which only guarantee truth-telling incentives for a *given* information structure.

Although this issue is already relevant in single-agent environments,[1] the degree of complexity is even higher in multi-agent environments: in principle, flexibility of each agent's information acquisition does not only mean flexibility in terms of his signal's informativeness about the payoff-relevant state, but also means flexibility in terms of his signal's informativeness about his opponents' signals. This issue of higher-order information and beliefs distinguishes multi-agent from single-agent environments. Modeling the dependence of the cost of information acquisition on higher-order information is a challenging task. In this paper, we assume that an agent's information acquisition cost only depends on his signal's informativeness about the payoff-relevant state, but not about the other agents' signals. In particular, it is costless to acquire a signal that is independent from the payoff-relevant state.[2] For example, imagine a situation where agents (e.g., telecom companies who buy spectrum) have to acquire information from data providers (e.g., market research firms) operating on a

[1] Mensch (2020) studies a mechanism design problem with a single agent. See also Section 1.1.

[2] The literature on cost of information proposes and discusses a variety of possible cost functions (see Section 1.1), but it seems to be universally accepted that uninformative signals about the state of the world are costless.

competitive market for data. Each data provider generates signals about the payoff-relevant state (e.g., demand conditions in the mobile services market). Competition among the data providers forces them to price their data at the cost of production, which in turn depends on the informativeness of their data. The agents can then decide to make their signals perfectly correlated by strategically choosing the same data provider, or less than perfectly correlated by choosing different data providers. In both cases, the agents will pay the same price for the same informativeness, and hence the cost of information acquisition will be independent of the correlation structure among signals.[3]

We consider a model with four or more agents. The principal and the agents share a common prior about the payoff-relevant state, and none of them has any private information at the beginning. We show that there exists a mechanism which allows the principal to implement any social choice rule at zero information acquisition cost to the agents. The key idea is that the mechanism recommends each agent to choose a special information acquisition action, which satisfies the *individually-uninformative-but-aggregately-revealing* property of Zhu (2021) (and each agent finds it optimal to obey this recommended action). The *individually-uninformative* part means that each agent's signal on its own is independent from the payoff-relevant state, which guarantees that his information cost is zero. The *aggregately-revealing* part means that the principal, by observing all the agents' reports — in fact, any two of them — can correctly identify the true payoff-relevant state. The fact that only two are enough, together with the fact that there are four or more agents, enables the principal to detect any unilateral deviation. It thus establishes the incentive compatibility of the mechanism.

1.1 Related Literature

In the literature on information acquisition in mechanism design, we usually consider restricted and/or less flexible spaces of information (see, for example, Bergemann and Välimäki (2002) for efficient mechanism design, Shi (2012) and Bikhchandani and Obara (2017) for optimal auction design, and Persico (2004), Gerardi and Yariv (2008), Gershkov and Szentes

[3]Of course, one can come up with cases where other cost specifications seem more reasonable (e.g., more positive correlation is more costly, or less costly). We discuss a range of possible alternative assumptions in our concluding remarks, see Section 5.

(2009), and Zhao (2016) for committee design with information acquisition[4]).

Mensch (2020) studies mechanism design with a single agent's flexible and costly information acquisition, building on the rational inattention framework (Sims (2003)).[5] Flexible and costly information acquisition is also considered by Gleyze and Pernoud (2020) who study a mechanism design problem with transferable utility and private values, in which agents acquire costly information on their own preferences and the preferences of other agents, and by Ravid et al. (2020) who study a bilateral trade model with costly information acquisition by the buyer. Flexible but not costly information acquisition is considered by Roesler and Szentes (2017) in the context of buyer-optimal information in monopoly pricing,[6] by Bergemann et al. (2017) and Brooks and Du (2021) in the context of seller-pessimal information in common-value auctions, and by Yamashita (2018) in private-value auctions. All these papers feature a single entity, "nature", who chooses the information structure (of one or multiple agents). In contrast to that, in our model each agent acquires information in a decentralized manner, which leads to a very different conclusion.

The information structure we employ was proposed in the context of mechanism design by Zhu (2021), who studies information disclosure by a mechanism designer. It builds on the idea of the one-time pad, an unbreakable encryption method (Shannon, 1949).[7]

This key information structure makes the agents' acquired information statistically dependent. In quasi-linear environments, Crémer and McLean (1988) show that the principal can extract full surplus from the agents who share a correlated prior. Although the extreme positivity of the results is a common feature of our paper and theirs, the two problems are

[4]Restricting to the class of conservative rules, Li (2001) solves for the optimal degree of conservatism in committee design. The optimally chosen conservative rule outperforms the *ex post* optimal rule.

[5]Mensch (2020) also considers a multiple-agent extension of his model, but restricts attention to symmetric mechanisms in an independent private values setting, in which agents can acquire information about their own values, but cannot acquire any information about others' values.

[6]See also Condorelli and Szentes (2020), though they also consider non-information changes of the agent's private information distribution.

[7]See also Krähmer (2020) and Krähmer (2021) in the context of information disclosure in mechanism design and strategic communication respectively where the randomization of information structures is allowed to keep the single agent (sender) uninformative; Kalai et al. (2010), Renou and Tomala (2012), Renault et al. (2014) in the context of games of communication network. Peters and Troncoso-Valverde (2013) apply this idea in mechanism-design games with multiple principals, and Liu (2015) applies it in his concept of individually uninformative correlating device. Our construction is most directly related to Zhu (2021).

quite different. First, our paper does not assume quasi-linearity. Second, their side-bet mechanism exploits an exogenously given correlated signal structure, and it is not clear if such a signal structure can be induced in equilibrium given some reasonable space of information acquisition actions.[8] In our case, the resulting information structure is an equilibrium outcome, even though each agent can potentially acquire information independently from the others' signals.

In non-quasi-linear environments, such as collective decision-making in committees, the first best outcome is generally not implementable under the commonly imposed restrictions on information acquisition technologies. For example, Li (2001) and Persico (2004), assuming that the agents have access to conditionally independent signals, show that the first best outcome is not attainable. In contrast to the previous results, we show that correlated information acquisition helps to implement the first best outcome.

There is a growing literature on the cost of flexible information in decision environments (see for example Sims (2003), Matejka and McKay (2015), Caplin and Dean (2015), and Pomatto et al. (2020)). Usually the main focus is on the cost of acquiring more or less precise information about a payoff-relevant state, and its relationship with a single decision-maker's optimal choice. The framework, however, has been applied in multi-player problems, e.g. in coordination games (Yang (2015); Morris and Yang (2021); Denti (2020)). In particular, Denti (2020) proposes a model of unrestricted information acquisition in games, in which, as in our paper, the players can endogenously learn about a payoff-relevant state and actions of other players.

2 Model

2.1 Setup

There is a principal and $I \geq 4$ agents, and a finite set of payoff-relevant states Θ. Each agent i's payoff is denoted $u_i(d, \theta)$, when a social decision $d \in D$ is selected in state θ.[9] For example, in an auction, d is a vector of bidders' winning probabilities and their expected

[8] Bikhchandani (2010) shows that, indeed, an agent in the Crémer-McLean mechanism may have a strong incentive of acquiring information about others.

[9] We can endow the principal with his own payoff function $u_0(d, \theta)$, though it is not necessary.

payments, and each u_i is quasi-linear in the payment part. Later, each agent's payoff *net* his information acquisition cost is considered as his objective.

At the beginning, neither the principal nor any of the agents know θ. The agents can acquire costly information about θ by generating private signals, possibly correlated with each other, whereas the principal cannot acquire any information about θ. Each agent has access to a sufficiently large set of possible signal realizations S_i. In principle, S_i (in particular, its size) may be a part of i's choice, but assuming exogenous S_i is without loss of generality as long as $|S_i| \geq |\Theta|$.

To model information acquisition, we introduce a space of states of nature $X = [0, 1]$ with a typical element x, equipped with a Borel σ-algebra and a uniform probability measure $\mathbb{P}$.[10] We assume there is a commonly known measurable function $\boldsymbol{\Theta} : X \to \Theta$ mapping the states of nature to the payoff-relevant states. This function induces a common prior on the payoff-relevant states as follows: $\mu_0(\theta) \equiv \int_0^1 1_{\{\boldsymbol{\Theta}(x)=\theta\}} dx$ for each $\theta \in \Theta$. Agent i's information acquisition action is a measurable function $\sigma_i : X \to S_i$, such that, once x (and hence $\theta = \boldsymbol{\Theta}(x)$) is realized, then i observes $s_i = \sigma_i(x)$. Let Σ_i denote the set of all such measurable functions, defining i's information acquisition action space. Note that any profile of information acquisition actions σ induces a joint distribution over payoff-relevant states and signal realizations, which we denote by $\alpha \in \Delta(\Theta \times S)$. When we want to make its dependence on σ more explicit, we write α_σ.

We assume that information acquisition is fully private in the sense that neither the principal nor any other agent observes which information acquisition action i takes and which signal realization is observed by agent i. Agent i's objective is the *net* payoff $u_i(d, \theta) - c_i(\sigma_i)$, where σ_i represents i's information acquisition action. We assume the information acquisition cost function of agent i has the following properties:

Assumption 1. Properties of information acquisition cost.

1. $c_i(\sigma_i) \geq 0$ *for any* σ_i.

2. $c_i(\sigma_i) = 0$ *if* σ_i *and* $\boldsymbol{\Theta}$ *are stochastically independent.*

[10] Taking a richer space of states of nature would not change our results. See also Gentzkow and Kamenica (2017) who use a similar approach in the context of multi-sender Bayesian persuasion.

The second property makes sure that agent i pays nothing as long as he learns nothing about the payoff-relevant state from his signal. This property is usually assumed in the context of single-player information acquisition.[11] For example, in the literature on rational inattention, the cost of information acquisition is often assumed to be proportional to the reduction in "relative entropy" (which measures the informativeness of a signal about the state). There, our second property is satisfied, because any signal that is stochastically independent from $\mathbf{\Theta}$ leaves the relative entropy unchanged, and is therefore costless.

With multiple players, even if a signal is uninformative about the payoff-relevant state, it could be informative about other players' signals, which is the key to our result. Our study can be interpreted as investigating the consequence of this assumption (seemingly quite natural in single-agent environments) in multi-agent mechanism design environments.

2.2 Mechanism

The principal faces both hidden action and hidden information of each agent. The principal commits to a mechanism at the *ex ante* stage in order to control the agents' incentives. More specifically, following the literature, we let the principal (i) send a message privately to each agent before his information acquisition action, and (ii) collect a message privately from each agent after the agent has observed a signal realization. Formally, a mechanism comprises $(R, \rho; M, \delta)$ where $R = (R_i)_{i=1}^I$ and $M = (M_i)_{i=1}^I$; R_i denotes the set of messages that the principal can send to each agent i; M_i denotes the set of messages that each agent i can send to the principal; $\rho \in \Delta(R)$ is a distribution over the principal's messages, and $\delta : R \times M \to D$ denotes the decision rule.

The timing of the game is summarized as follows:

$t = 0$: $x \sim U(0, 1)$ is drawn but no one observes it.

$t = 1$: The principal designs a mechanism $(R, \rho; M, \delta)$.

$t = 2$: After observing the mechanism and receiving $r_i \in R_i$, each agent i privately chooses his information acquisition action $\sigma_i \in \Sigma_i$.

[11] See the literature on cost of information, such as Sims (2003), Matejka and McKay (2015), Caplin and Dean (2015), and Pomatto et al. (2020).

$t = 3$: Each agent i privately observes $s_i = \sigma_i(x)$, and privately sends $m_i \in M_i$ to the principal.

$t = 4$: The principal executes $d = \delta(r, m)$ where $m = (m_i)_{i=1}^I$.

Because no agent observes the other agents' actions or information (even noisily) at all, we consider Nash equilibrium as a solution concept. Then, applying the revelation principle of Forges (1986), we focus on *direct* mechanisms where (i) the principal directly recommends an information-acquisition action to each agent, and each agent directly reports a signal to the principal, and (ii) each agent finds it optimal to obey the recommended action and truthfully report his signal.[12]

Formally, a direct mechanism comprises $((\sigma_i)_{i=1}^I, (S_i)_{i=1}^I, \delta)$, where the principal recommends $\sigma_i \in \Sigma_i$ privately to each agent i,[13] and executes $\delta(s) \in D$ if the agents report $s = (s_i)_{i=1}^I \in S = \times_{i=1}^I S_i$. A direct mechanism is *incentive compatible* if it satisfies the following constraints: for any $\sigma_i' \in \Sigma_i$ and $\tau_i : S_i \to S_i$,

$$\sum_{\theta, s_i, s_{-i}} (u_i(\delta(s_i, s_{-i}), \theta)\alpha_{\sigma_i, \sigma_{-i}}(\theta, s_i, s_{-i})) - c_i(\sigma_i) \geq \sum_{\theta, s_i, s_{-i}} (u_i(\delta(\tau_i(s_i), s_{-i}), \theta)\alpha_{\sigma_i', \sigma_{-i}}(\theta, s_i, s_{-i})) - c_i(\sigma_i').$$

That is, each i must find it optimal to obey the recommended σ_i and report the realized s_i truthfully.

Although the constraints are concisely summarized by the inequalities above, they are actually rather complicated. First, changing σ_i affects the joint distribution α of (θ, s) and the agent's cost in a non-trivial way since agent i cannot affect agent $-i$'s information structure. Second, an agent may potentially want to make a double deviation, that is, change σ_i and at the same time change his reporting strategy.

Remark 1. *Here, we do not explicitly impose individual rationality constraints. It is not difficult to accommodate these constraints: let us require that any feasible direct mechanism*

[12] The proof proceeds as follows. First, imagine an auxiliary game where there is no principal, but instead, there is a fictitious player ("player 0") who is indifferent across all decisions in any state. At first, each agent i plays σ_i privately, and then observes the realized signal s_i privately. Then, (without any communication), player 0 chooses $d \in D$. Interpreting this as a baseline extensive-form game, it is easy to see that our current game (with the principal) is the mediated communication game of this auxiliary game in the sense of Forges (1986) (see also Myerson (1986)). Thus, her revelation principle applies.

[13] We focus on a deterministic recommendation of σ, rather than any stochastic recommendation. Accordingly, δ is denoted simply by $\delta(s)$ instead of $\delta(r, s)$. Since first best implementation is achieved with pure recommendations, our focus on them is without loss of generality.

must have an extra message $m_i^{\emptyset}$ (a "non-participation" message) so that i's message space is now $S_i \cup \{m_i^{\emptyset}\}$, and $\delta(m_i^{\emptyset}, m_{-i})$ is some specific allocation (a "non-participation allocation") for agent i, for any given m_{-i}. When the non-participation message is included into the set of messages for each agent, the individual rationality constraints, both at the ex ante and interim stages, are captured by the above incentive compatibility constraints.[14]

3 Main result

Fix any function $d^* : \Theta \to D$, which describes all the economically relevant outcomes in this environment except for the information acquisition costs. If the principal could observe θ, then any d^* is attainable without any information acquisition cost on the agents' side. In this sense, one may interpret this d^* together with zero cost for the agents as the *first-best* outcome.[15]

In this section, for any given d^*, we explicitly construct a mechanism that implements d^* at zero cost for the agents. That is, the first best outcome can be attained *even though the principal cannot directly observe θ.*

Theorem 1. *Fix any $d^* : \Theta \to D$. Under Assumption 1, there exists a mechanism (σ, S, δ) such that (i) $\sum_s \alpha(\theta, s) 1_{\{\delta(s)=d^*(\theta)\}} = \mu_0(\theta)$ for all θ, and (ii) $c_i(\sigma_i) = 0$ for all i.*

Proof. The theorem is proved by construction.

Since Θ is a finite set, we assume without loss of generality that $\Theta = \{1, \ldots, T\}$. Let $K > \max\{I, T\}$ be a prime number. Because $\mathbb{P}$ is a uniform measure on $X = [0, 1]$, we can find a partition of X, denoted by $\{X_{\theta\psi}\}_{(\theta,\psi)\in\{1,\ldots,T\}\times\{1,\ldots,K\}}$, satisfying $\int_0^1 1_{\{x \in X_{\theta\psi}\}} dx = \frac{1}{K}\mu_0(\theta)$ for any θ and ψ. Define a measurable function $\mathbf{\Psi} : [0, 1] \to \{1, \ldots, K\}$ such that, if $x \in \cup_{\theta\in\Theta} X_{\theta\psi}$, then $\mathbf{\Psi}(x) = \psi$. Immediately, $\mathbf{\Psi}$ is uniformly distributed on $\{1, \ldots, K\}$ conditional on any realization θ of $\mathbf{\Theta}$, hence $\mathbf{\Psi}$ is independent of $\mathbf{\Theta}$.

Now consider the following information acquisition action profile: for each $i \in \{1, \ldots I\}$, $S_i = \{1, \ldots, K\}$, and $\sigma_i(x) = \mathbf{\Theta}(x) + i \cdot \mathbf{\Psi}(x) \mod K$ for any $x \in [0, 1]$. Note that the

[14] Ex interim individual rationality is guaranteed because agent i can always deviate to $\tau_i(\cdot) \equiv m_i^{\emptyset}$. Ex ante individual rationality is guaranteed because agent i can always deviate to a costless σ_i and then to $\tau_i(\cdot) \equiv m_i^{\emptyset}$.

[15] For example, one may assume that $d^*(\theta)$ is the best decision of the principal given his own preferences in state θ.

residual is calculated as in standard modular arithmetic except when $\mathbf{\Theta}(x) + i \cdot \mathbf{\Psi}(x)$ is divisible by K, in which case we set $\sigma_i(x) = K$ instead of 0. The following lemma gives the properties of (S, σ) that we need to prove the theorem.

Lemma 1. *The above (S, σ) satisfies:*

(i) For any $i \in \{1, \dots, I\}$, σ_i is independent of $\mathbf{\Theta}$.

(ii) Conditional on any realization of (s_i, s_j) such that $i \neq j$, the joint distribution of $\mathbf{\Theta}$ and $(\sigma_k)_{k \neq i,j}$ is degenerate.

Proof of the lemma. By definition of (S, σ), for each i, we have $s_i = \theta + i \cdot \psi \mod K$, where the random variables $\mathbf{\Psi}$ and $\mathbf{\Theta}$ are independent. Thus the signal profile $s = (s_i)_{i=1}^{I}$ is defined in the same way as in Zhu (2021).[16] Thus, this lemma is directly implied by Lemma 2 in Zhu (2021). $\square$

The first property says that $\mathbf{\Theta}$ and σ_i are independent, implying $c_i(\sigma_i) = 0$. The second property says that, given s_i, s_j with $i \neq j$, we can identify the true payoff-relevant state θ and any signal realization s_k without error, that is, there exist $\hat{\theta}(s_i, s_j)$ and $\hat{s}_k(s_i, s_j)$ such that:

$$\Pr(\mathbf{\Theta} = \hat{\theta}(s_i, s_j) | s_i, s_j) = \Pr(\sigma_k = \hat{s}_k(s_i, s_j) | s_i, s_j) = 1.$$

Let the principal recommend the above σ, and offer the decision rule δ as follows: $\delta(s) = d^*(\theta)$ if (i) for any i, j with $i \neq j$, we have

$$\theta = \hat{\theta}(s_i, s_j);$$

or if (ii) there is i such that, for any j, k where i, j, k are all different, we have

$$\theta = \hat{\theta}(s_j, s_k).$$

In any other case, $\delta(s)$ is arbitrary.

Clearly, if the agents obey the recommendation and report their signals truthfully, then the first best outcome is attained. Therefore, we complete the proof by showing that the

[16] In fact, our signal profile s coincides with what Zhu (2021) calls "the IUAR disclosure policy, where IUAR is short for *individually uninformative but aggregately revealing*.

proposed mechanism satisfies incentive compatibility. Take any agent i, and suppose that he deviates to any σ_i' and reports $\tau_i(s_i)$ when s_i is realized. First, his cost of information acquisition increases weakly. Second, his reporting decision does not affect the social decision at all, because the principal executes $\delta(s) = d^*(\hat{\theta}(s_j, s_k))$ for an arbitrary pair (j, k) which does not include i. Therefore, the mechanism is incentive compatible. $\square$

3.1 Impossibility results with two and three agents

One could ask whether a result similar to Theorem 1 obtains with two or three agents. The general answer to this question is *no*. With three agents, although it is possible to determine whether *some* agent has unilaterally deviated or not, it is not possible to identify who the deviator is (and hence not possible to identify the true θ). To see that, consider the following counterexample.

Counterexample 1. *Suppose that there are two payoff-relevant states, i.e.* $\Theta = \{1, 2\}$ *and consider the mechanism with* $K = 5$. *Computing* $s_i = \theta + i\psi \mod 5$, *we obtain:*

$\theta = 1$	Agent 1	Agent 2	Agent 3
$\psi = 1$	2	3	4
$\psi = 2$	3	5	2
$\psi = 3$	4	2	5
$\psi = 4$	5	4	3
$\psi = 5$	1	1	1

$\theta = 2$	Agent 1	Agent 2	Agent 3
$\psi = 1$	3	4	5
$\psi = 2$	4	1	3
$\psi = 3$	5	3	1
$\psi = 4$	1	5	4
$\psi = 5$	2	2	2

Suppose the principal observes an out-of-equilibrium signal realization profile $(2, 5, 4)$. *There are two unilateral deviations that lead to this profile. First, the true profile might be* $(2, 3, 4)$ *in state* $\theta = 1$ *with agent 2 deviating. Second, the true profile might be* $(1, 5, 4)$ *in state* $\theta = 2$ *with agent 1 deviating. Hence, the principal cannot identify the deviator, nor can the principal infer the true state.*

Note, however, that if there exists a social decision $d \in D$ that can serve as a severe punishment for all agents for any given θ, then out-of-equilibrium reports can be severely punished by the principal,[17] and a similar first-best implementation result obtains.

[17] Consider e.g. environments with monetary transfers, in which the principal can use large fines to punish agents for inconsistent reports.

With two agents, each agent has much more freedom. The authors work on a separate project with two agents. There, even under Assumption 1 and even when monetary transfers are available to the principal, an extremely positive result similar to Theorem 1 does not generally hold. The optimal mechanism might involve some costly information acquisition.

4 Applications

4.1 Full-surplus extraction in common value auctions

Consider the following common value auction environment. The seller (principal) has a single indivisible good, and there are $I \geq 4$ bidders. The value of the good is common to all the bidders, denoted by $\theta \in \Theta$, where Θ is finite. In fact, the analysis of this section can be straightforwardly extended to the case of "non-pure" common values where each i's valuation is $v_i(\theta)$. Let $\mu_0(\theta)$ denote the probability that θ is the bidders' common value.

Each bidder i's payoff is $\theta q_i - t_i - c_i(\sigma_i)$ if he wins the good with probability q_i, pays t_i to the seller, and spends $c_i(\sigma_i)$ as his information acquisition cost. In case he does not participate in the mechanism, his outside-option payoff is 0. The seller's payoff is revenue, $\sum_{i=1}^{I} t_i$.

The first-best expected surplus of this society is the expected common value:

$$\sum_{\theta \in \Theta} \mu_0(\theta)\theta = \mathbb{E}[\theta].$$

There are several cases where the seller can easily earn $\mathbb{E}[\theta]$. First, if the seller *knows* θ, then he can simply post price θ. Even if the seller does not know θ, if the bidders know θ as their common knowledge (i.e., as *free* information), then again the seller can earn $\mathbb{E}[\theta]$. Conversely, if all the bidders are completely *uninformed* (so that each only knows the common prior μ_0), then again, the seller can post price $\mathbb{E}[\theta]$.

Notice that, with costly information acquisition as considered in our paper, neither of the above ideas would work. First, although it might be possible to make every bidder fully learn θ in some equilibrium, it does not yield $\mathbb{E}[\theta]$ as long as full information is strictly costly. Second, if the seller posts price $\mathbb{E}[\theta]$, then each bidder has a strong incentive of knowing whether the true θ is below $\mathbb{E}[\theta]$ or not: If i finds that $\mathbb{E}[\theta|s_i] < \mathbb{E}[\theta]$ given some signal s_i, he would not buy the good. As long as such information is not too costly, the bidder would be better off by acquiring it.

Therefore, with a general information acquisition cost function, the equilibrium information should be somewhere between full and no information, and it is *a priori* unclear how the seller should find the optimal balance of information and rent extraction. Nevertheless, as long as the cost functions satisfy Assumption 1, Theorem 1 implies that the full-surplus extraction is possible.

Corollary 1. *Under Assumption 1, there is a mechanism which yields $\mathbb{E}[\theta]$ as the seller's expected revenue (and each bidder earns 0).*

It is worth emphasizing that the logic here is very different from that of Crémer and McLean (1988). In their paper, the seller exploits an *exogenously given* correlated signal structure, in order to construct a side-bet scheme that extracts the entire surplus. In our case, each bidder can choose any information structure. Indeed, if he prefers, a bidder can choose an information structure such that his information is *independent* from all the other bidders' signals (conditional on the state of the world). The Crémer-McLean lottery scheme, therefore, does not work here. Also, in their auction, each bidder's payoff can be strictly negative *ex post*, while in our case, it is zero *ex post*. Indeed, if the seller offered a negative *ex post* payoff in our auction, bidders would have a strong incentive to get a signal which includes a realization indicative of that event and then abstain from the auction following that realization.

4.2 First-best implementation in collective decision-making

Consider a committee with a designer (principal) and $I \geq 4$ members (agents) deciding whether to hire or not to hire a job market candidate. Formally, $d \in D = \{h, nh\}$. The quality of the candidate is $\theta \in \Theta$, which is unobserved *ex ante*. The designer and all members of the committee hold a common prior belief $\mu_0 \in \Delta(\Theta)$ about the candidate's quality.

The utility that each member obtains from hiring / not hiring the candidate is defined as follows:

$$u_i(d, \theta) = \begin{cases} u_i(\theta), & \text{if } d = h \\ 0, & \text{if } d = nh \end{cases}$$

Without loss of generality, we assume that $u_i(\theta) = k_i\theta$.

Only the committee members can acquire information about the candidate at cost $c_i(\sigma_i)$. The designer aims to maximize the expected sum of all members' gross utilities.[18] That is, ideally, he wants to hire the candidate if and only if $\sum_i k_i\theta \geq 0$. The first best expected surplus of all committee members is given by:

$$\sum_{\theta | \sum_i k_i\theta \geq 0} \mu_0(\theta) \sum_i k_i\theta \equiv W^{FB}$$

It is useful to note that the existing literature (see e.g. Li (2001) and Gerardi and Yariv (2008)) typically assumes that the committee members have access to information structures whose realized signals are independently distributed across them, conditional on the state of the world. Under these restrictions, the first best outcome cannot be implemented. There are two main forces that prevent the committee from implementing the first best with these restricted information structures: free-riding problem and conflict of interest. First, when committee members have a conflict of interest, they may prefer not to report their own acquired information truthfully. Second, even if all members share a common preference, information could be underprovided relative to the social optimum, because it is essentially a public good used to make a collective decision. For example, Li (2001) suggests that distorting the decision rule away from the *ex post* optimal rule (which is optimal under exogenous information) could help to alleviate the free-riding issue.

In contrast to the previous literature, our results show that with more flexible (even though still costly) information acquisition the designer can implement the first best outcome. On the one hand, having access to a wider range of information acquisition technologies enlarges the set of feasible deviations for the agents. On the other hand, the principal now has more flexibility in designing information structures recommended to the agents. Given these two opposing effects, it is not immediately clear *a priori* whether the first best outcome becomes more or less difficult to attain. It turns out that the second effect dominates: with a larger set of feasible mechanisms, the principal is able to incentivize the agents to acquire and report their information truthfully, no matter what social choice rule the principal is trying to implement. Indeed, Theorem 1 implies that the first best is implementable as long

[18]The result extends to the case where the designer maximizes expected sum of members' net utilities (taking into account the information acquisition costs).

as Assumption 1 about the cost functions holds.

Corollary 2. *Under Assumption 1, there is a mechanism which yields W^{FB} as the total expected surplus (the decision is made under full information with no cost).*

Our construction helps to resolve both of the issues that prevent first best implementation with conditionally independent signals. Recall that it costs nothing for an agent to acquire an *"individually uninformative"* signal which is assigned to him under the optimal mechanism. Therefore, the distorted provision of information is resolved. Moreover, even if committee members have a conflict of interest, under our mechanism, they cannot do better than being truthful since any unilateral deviation can be detected by the designer.

5 Concluding remarks

It is quite natural that agents may desire to refine their information in response to a mechanism. This paper proposes one possible framework, based on a class of information acquisition cost functions, such that the cost of information depends on the informativeness of each agent's signal about the state of the world, but not on its informativeness about other agents' signals. We show that such a specification leads to an extremely positive result.

One natural criticism may be that our mechanism induces full information if the signals are aggregated, even though any single signal is completely uninformative: would it be reasonable to assume that such σ_i is costless? Because the answer is necessarily yes under Assumption 1, the question is essentially whether Assumption 1 itself is reasonable. Assumption 1 is satisfied, in particular, in any information acquisition environment, in which the cost of information acquisition does not depend on the correlation structure among signals but only depends on their individual informational content. We argued in Section 1 that there are information acquisition environments, for which this assumption is indeed a reasonable one. In general, however, the correlation structure might affect the cost of information acquisition in various ways. On the one hand, there seem to be cases where more positive correlation is more expensive. For example, fix agent 1's private information, and consider agent 2. If acquiring a positively correlated information necessarily means that agent 2 must *steal* (perhaps a part of) agent 1's information, more positive correlation will be more costly. Strulovici (2021), for instance, considers an environment where hard evidence is scarce in the sense that, if one agent "picks up" a piece of evidence, then it becomes difficult for the others to get the same or similar evidence. On the other hand, there are opposite situations, where *less correlated* signals are more costly. For example, suppose that there are 2 agents and 3 newspapers, and σ_i corresponds to the decision of which newspapers to buy. Suppose further that newspaper 3 is free, hence both agents read it and acquire perfectly correlated signals for free. To acquire less correlated signals, at least one of the agents has to buy additional information (e.g. agent 1 buys newspaper 1 and/or agent 2 buys newspaper 2), hence less correlation can be more costly in this example.

This discussion suggests that we must think more about modeling the microstructure of information acquisition, in order to determine which information structures are more costly. Mechanism design with such more specific information acquisition cost structures would certainly be an interesting future direction, and we hope this article could serve as a first step in that direction.

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